Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel GCE In A level Further Mathematics
Paper 9FM0/3D

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

9FM0/3D: Decision Mathematics D1 Mark scheme

Question	Scheme	Marks	AOs
1(a)	Bin 1: <u>2.1</u> <u>1.7</u> <u>1.2</u> Bin 2: <u>3.0</u> <u>1.9</u> Bin 3: <u>3.2</u> 1.4 0.2 Bin 4: 3.3 1.5	<u>M1</u> A1	1.1b 1.1b
		(2)	
(b)	e.g. middle right 2.1 1.7 3.0 1.9 3.2 $\boxed{1.2}$ 3.3 1.4 1.5 0.2 Pivot: 1.2 2.1 1.7 3.0 1.9 $\boxed{3.2}$ 3.3 1.4 1.5 <u>1.2</u> 0.2 Pivot(s): 3.2 (0.2) 3.3 <u>3.2</u> 2.1 1.7 3.0 $\boxed{1.9}$ 1.4 1.5 <u>1.2</u> 0.2 Pivot(s): (3.3) 1.9 3.3 <u>3.2</u> 2.1 $\boxed{3.0}$ <u>1.9</u> 1.7 $\boxed{1.4}$ 1.5 <u>1.2</u> 0.2 Pivots: 3.0 1.4 3.3 <u>3.2</u> <u>3.0</u> 2.1 <u>1.9</u> 1.7 $\boxed{1.5}$ <u>1.4</u> <u>1.2</u> 0.2 Pivot(s): (2.1) 1.5 3.3 <u>3.2</u> <u>3.0</u> 2.1 <u>1.9</u> 1.7 <u>1.5</u> <u>1.4</u> <u>1.2</u> 0.2	M1 A1 A1ft A1	1.1b 1.1b 1.1b 1.1b
		(4)	
(c)	$\dfrac{2.32(11\,250\log 11\,250)}{450\log 450}$ =88.6 seconds	M1 A1	1.1a 1.1b
		(2)	
			(8 marks)

Notes for Question 1

PLEASE NOTE NO MISREADS IN PARTS (a) and (b) – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE FOR ASCENDING ORDER IN (b)

(a) M1: First six items placed correctly and at least eight values placed in bins - condone cumulative totals for M1 only (the underlined values)
A1: CSO – all correct (so no additional/repeated values)

(b) M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then max of M1A1 only** – Bubble sort is not a MR and scores M0
A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct) – so they must be choosing (if middle right) a pivot value of 3.2 for the second pass or (if middle left) a pivot value of 1.9
A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark
A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including if middle right a fifth pass with the 1.5 used as a pivot or if middle left a fourth pass with the 1.7 used as a pivot

Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in **(b)**
- If the list is not reversed in **(b)** then remove the last two A marks earned in **(b)**. If the candidate says that the list needs reversing in **(b)** but does not actually show the reversed list in **(b)** then remove the last A mark earned
- **Note that if sorting into ascending order then a 'sort complete' statement is required – this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice) BEFORE list is reversed**

Middle left

2.1	1.7	3.0	1.9	3.2	1.2	3.3	1.4	1.5	0.2
3.3	3.2	2.1	1.7	3.0	1.9	1.2	1.4	1.5	0.2
3.3	3.2	2.1	3.0	1.9	1.7	1.2	1.4	1.5	0.2
3.3	3.2	3.0	2.1	1.9	1.7	1.5	1.4	1.2	0.2
3.3	3.2	3.0	2.1	1.9	1.7	1.5	1.4	1.2	0.2

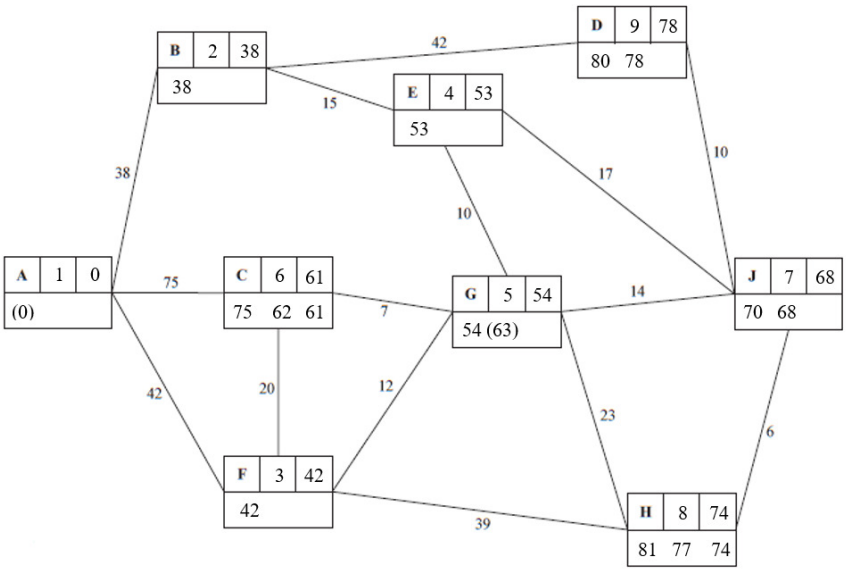
Middle right ascending (which requires a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	3.2	1.2	3.3	1.4	1.5	0.2
0.2	1.2	2.1	1.7	3.0	1.9	3.2	3.3	1.4	1.5
0.2	1.2	2.1	1.7	3.0	1.9	1.4	1.5	3.2	3.3
0.2	1.2	1.7	1.4	1.5	1.9	2.1	3.0	3.2	3.3
0.2	1.2	1.4	1.7	1.5	1.9	2.1	3.0	3.2	3.3
0.2	1.2	1.4	1.5	1.7	1.9	2.1	3.0	3.2	3.3

Middle left ascending (which required a 'sort complete' statement – see above)

2.1	1.7	3.0	1.9	3.2	1.2	3.3	1.4	1.5	0.2
2.1	1.7	3.0	1.9	1.2	1.4	1.5	0.2	3.2	3.3
1.7	1.2	1.4	1.5	0.2	1.9	2.1	3.0	3.2	3.3
1.2	0.2	1.4	1.7	1.5	1.9	2.1	3.0	3.2	3.3
0.2	1.2	1.4	1.5	1.7	1.9	2.1	3.0	3.2	3.3

(c) M1: Complete correct method – allow reciprocal – allow slips in values only e.g. 1250 for 11 250
A1: CAO – the exact value of 88.6 must be stated at some point (as question specifically asked for the answer to the nearest tenth of a second) – isw if 90 follows 88.6 seen. 90 with no working scores no marks. An answer of 88.6 with no working scores M1A0 – condone lack of units (but if present must be correct)

Qu	Scheme	Marks	AOs
2(a)		M1	1.1b
		A1	1.1b
		A1	1.1b
		A1ft	1.1b
	Path from A to D is AFGJD	A1	2.2a
	Length of path from A to D is 78 metres	A1ft	2.2a
		(6)	
(b)		M1	3.1b
	A(FG)C + E(J)H = 61 + 23 = 84 A(B)E + C(GJ)H = 53 + 27 = 80* A(FGJ)H + C(G)E = 74 + 17 = 91	A1ft A1	1.1b 1.1b
	Repeat arcs: AB, BE, CG, GJ and JH	A1	2.2a
		(4)	
(c)	Length of the route is 370 + 80 = 450 metres	B1ft	2.2a
		(1)	
(d)(i)	If node B is removed this makes D, C, G and H odd	M1	3.1b
	The shortest path between any two odd nodes is CG (so repeat CG) so the route should start at D and finish at H (or vice-versa)	A1	2.2a
(ii)	Length of new route is 370 – 38 – 42 – 15 + 7 = 282 metres	B1	2.2a
		(3)	
			(14 marks)

Notes for Question 2
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 81 77 74 in that order (so 81 74 77 is incorrect) **It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling** **(a) M1:** A larger value replaced by a smaller value in at least two of the working boxes at either C or D or E or G or J **A1:** All values in A, B, F and E correct. Condone lack of 0 in A's working value **A1:** All values G, C and J correct and the working values in the correct order. Penalise order of labelling only once per question (G, C and J must be labelled in that order and G must be labelled after A, B, F and E). Note that an additional working value of 63 at G after the 54 is not an error so 54 63 is fine, however, any other number or 63 54 in this order is incorrect and scores A0 in this part **A1ft:** All values in H and D correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through H check that the working value at H follows from the candidate's final values from their feeds into H (which will come from nodes F, G and/or J (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for D (which will possibly have working values from B and J with the order of these values determined by the candidate's order of labelling at B and J) **A1**: CAO - correct path from A to D (AFGJD) **A1ft:** ft their final value at D only (if 78 stated and 78 is not the final value at D then A0) **(b) M1:** correct three pairings of the correct four odd nodes (A, C, E and H) **A1ft:** any row correct including pairing **and** total (ft the final values from **(a)** for their shortest paths from A to the three other nodes C, E or H only (so the pairing that does not include A must be correct)) **A1:** all three rows correct including pairings **and** totals **A1:** selecting the shortest pairing, and stating that these arcs (AB, BE, CG, GJ and JH) should be repeated. Must be these arcs and not e.g. ABE, CGJH or AE via B, etc. **(c) B1ft:** For 370 + their smallest repeat out of a choice of at least **two** totals seen in **(b)** – this mark is dependent on M1 in (b) **(d) M1:** Mention of the fact that these four nodes D, C, G and H only are now odd **or** clear consideration of these four nodes only **A1:** CAO D and H **and** must have clearly indicated that the shortest path is from C to G (or vice-versa) – but A0 if clearly selecting the shortest pairing first before selecting the shortest path (as the shortest path is embedded in the shortest pairing) **B1**: CAO (282)

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>3(a)</td><td>Distance table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>15</td><td>7</td><td>18</td><td>3</td></tr>
<tr><td>B</td><td>15</td><td>-</td><td>∞</td><td>∞</td><td>∞</td></tr>
<tr><td>C</td><td>7</td><td>5</td><td>-</td><td>4</td><td>9</td></tr>
<tr><td>D</td><td>18</td><td>∞</td><td>4</td><td>-</td><td>3</td></tr>
<tr><td>E</td><td>∞</td><td>∞</td><td>9</td><td>3</td><td>-</td></tr>
</table>
Route table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
</table></td><td>B1

B1</td><td>1.1b

1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(b)</td><td>1st iteration:
Distance table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>15</td><td>7</td><td>18</td><td>3</td></tr>
<tr><td>B</td><td>15</td><td>-</td><td>22</td><td>33</td><td>18</td></tr>
<tr><td>C</td><td>7</td><td>5</td><td>-</td><td>4</td><td>9</td></tr>
<tr><td>D</td><td>18</td><td>33</td><td>4</td><td>-</td><td>3</td></tr>
<tr><td>E</td><td>∞</td><td>∞</td><td>9</td><td>3</td><td>-</td></tr>
</table>
Route table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>A</td><td>A</td><td>A</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>A</td><td>A</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
</table></td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>2nd iteration:
Distance table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>15</td><td>7</td><td>18</td><td>3</td></tr>
<tr><td>B</td><td>15</td><td>-</td><td>22</td><td>33</td><td>18</td></tr>
<tr><td>C</td><td>7</td><td>5</td><td>-</td><td>4</td><td>9</td></tr>
<tr><td>D</td><td>18</td><td>33</td><td>4</td><td>-</td><td>3</td></tr>
<tr><td>E</td><td>∞</td><td>∞</td><td>9</td><td>3</td><td>-</td></tr>
</table>
Route table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>A</td><td>A</td><td>A</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>A</td><td>A</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
</table></td><td>A1ft</td><td>1.1b</td></tr>
<tr><td></td><td>3rd iteration (SEE NOTES FOR VALID ALTERNATIVE):
Distance table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>12</td><td>7</td><td>11</td><td>3</td></tr>
<tr><td>B</td><td>15</td><td>-</td><td>22</td><td>26</td><td>18</td></tr>
<tr><td>C</td><td>7</td><td>5</td><td>-</td><td>4</td><td>9</td></tr>
<tr><td>D</td><td>11</td><td>9</td><td>4</td><td>-</td><td>3</td></tr>
<tr><td>E</td><td>16</td><td>14</td><td>9</td><td>3</td><td>-</td></tr>
</table>
Route table
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>A</td><td>C</td><td>C</td><td>C</td><td>E</td></tr>
<tr><td>B</td><td>A</td><td>B</td><td>A</td><td>C</td><td>A</td></tr>
<tr><td>C</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>D</td><td>C</td><td>C</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>E</td><td>C</td><td>C</td><td>C</td><td>D</td><td>E</td></tr>
</table></td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(c)(i)</td><td>Start at E(5th row) and read across to the A (1st column), there is a D there so the route from E to A is via D</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>Now consider both E to D and D to A – for E reading across to the D (4th column), there is a D indicating that the shortest path from E to D is ED. For D reading across to the A (1st column), there is a C indicating that the shortest path from D to A is via C</td><td>dB1</td><td>2.4</td></tr>
<tr><td>(ii)</td><td>EDCA</td><td>B1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
</table>

(d)(i)	NNA: D – E – C – B – A – D	B1	1.1b
(ii)	3 + 7 + 5 + 15 + 6 = 36 miles	B1	1.1b
(iii)	D – E – D – C – B – A – E – D	B1	3.2a
(iv)	e.g. the cycle A – E – D – C – B – A has a length of 30 miles < 36 miles so Mabintou's route is not optimal	B1	2.4
		(4)	
			(14 marks)

Notes for Question 3

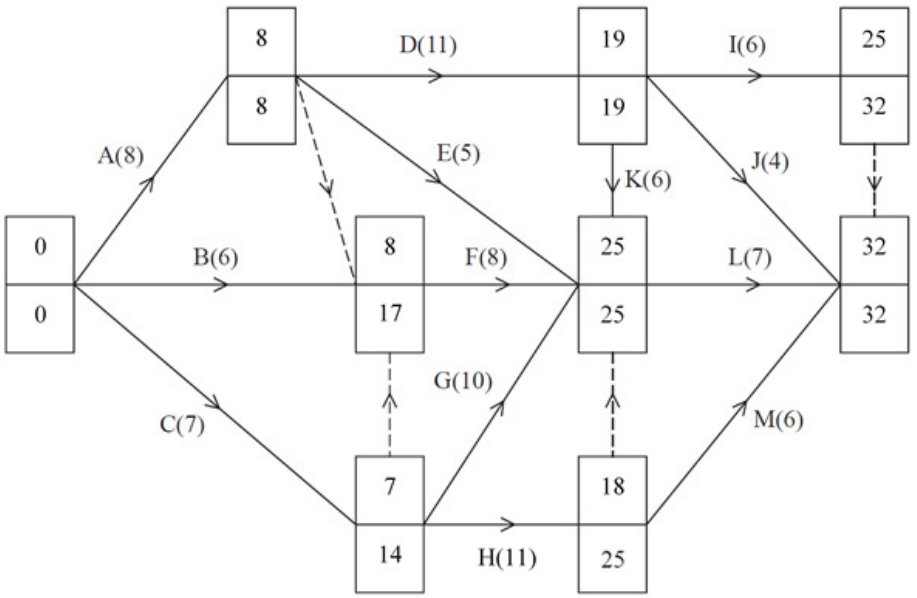
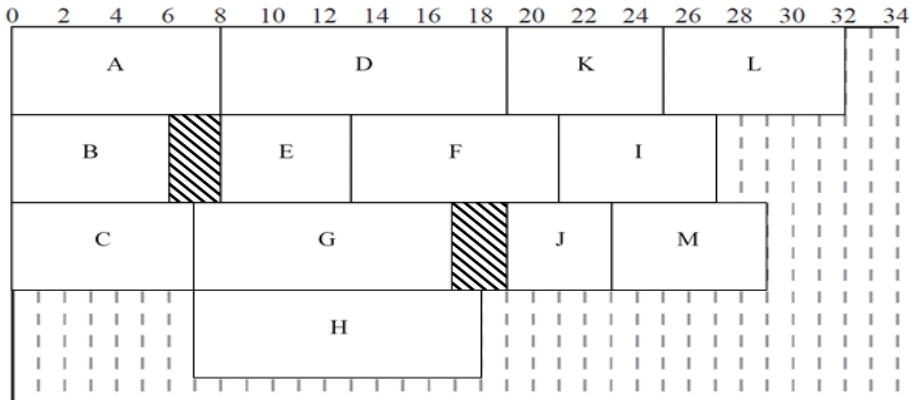
IN THE DISTANCE AND ROUTE TABLE FOR PARTS (a) and (b) IGNORE WHATEVER IS WRITTEN IN THE LEAD DIAGONAL (TOP LEFT TO BOTTOM RIGHT)

(a) B1: Correct distance table (condone dashes, crosses, etc. for infinity but do not condone a 'large' number in these cells)
B1: Correct route table

(b) M1: No change in the first row and first column of both tables with at least two values in the distance table correctly reduced and two letters in the route table correctly changed – all cells complete
A1: CAO (condone dashes, etc. in cells EA and EB)
A1ft: No change from candidate's first iteration to second iteration for either table or ft from candidate's first iteration
M1: No change in the third row and third column of both tables with at least two values in the distance table correctly reduced from their second iteration and two values in the route table correctly changed
A1: CAO for third iteration (**note that the entry in row B column D for the route table could be an A**)

(c) B1: Row E column A is D so the route is E to A via D (or implies that the order of the nodes in the route is EDA) **or** D implied from general argument **or** Row E column A is D therefore the route begins ED (in all cases must clearly imply **row** E and **column** A)
dB1: Row D column A is C therefore the route goes via C (before A) **or** complete general argument that allows the route from D to A to be found **or** allow those who say that row D column A is C so the route is EDC and then row C column A is A
B1: CAO (EDCA)

(d)(i) B1: CAO (D – E – C – B – A – D)
(ii) B1: CAO (36 – no units required)
(iii) B1: CAO (D – E – D – C – B – A – E – D) **or** mentions that the cycle would visit E twice and D three times (or visit D before the end of the cycle – if D visited once stated and it is not clear that this isn't the start or finish then B0) **or** mention of E to C via D and A to D via E
(iv) B1: A correct cycle stated (e.g. a cyclic permutation of A – E – D – C – B – A) with corresponding correct length – **dependent on second B mark in this part** (so must have had 36 in **(d)(ii)**)

Qu	Scheme	Marks	AOs
4(a)	The early event time at the end of activity C is 7 (as no other activity leads into this event). Therefore the float on activity H is $25 - 7 - x$ The float on activity H is given as 7 and so therefore $25 - 7 - x = 7$ implies that the value of x is equal to $25 - 7 - 7 = 11$	B1 dB1	3.1a 2.4
		(2)	
(b)		M1 A1 A1	2.1 1.1b 1.1b
		(3)	
(c)	$\frac{95}{32} = 2.968... = 3$ workers	B1	2.2a
		(1)	
(d)	e.g. 	M1 A1 A1	2.1 1.1b 1.1b
		(3)	
			(9 marks)

Notes for Question 4

(a) B1: correct reasoning for why the float on activity H is given by $25 - 7 - x$, must mention that the early event time at the end of activity C is 7 or the early event time at the start of H is 7 **and** that the total float for H is therefore $25 - 7 - x$ (or $25 - x - 7$ but not just $18 - x$) (no reason for why the early event time at the end of C is 7 is required)

dB1: correct explanation for why $x = 11$ (dependent on previous B mark) – as a minimum must equate $25 - 7 - x$ to 7 (allow $18 - x = 7$ as they must have shown where the 18 comes from to get the first B mark) and hence $x = 11$

SC B1B0 – for those who write or imply $25 - 7 - x = 7$ (but not just $18 - x = 7$) and state $x = 11$ without any mention of the early event time at the end of C or the total float of activity H. However, $25 - 7 - 7 = 11$ only is no marks in this part

(b) M1: All top boxes and all bottom boxes completed. Values generally increasing left to right (for top boxes) and values generally decreasing from right to left (for bottom boxes). Condone missing 0s at the source node or the 32 in the bottom box at the sink node for M only. Condone one rogue value in top boxes and one rogue value in bottom boxes. For a rogue in the top boxes if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value (with a similar definition for bottom boxes but in reverse)

A1: CAO - Top boxes (including zero at the source node)

A1: CAO - Bottom boxes (including zero at the sink node)

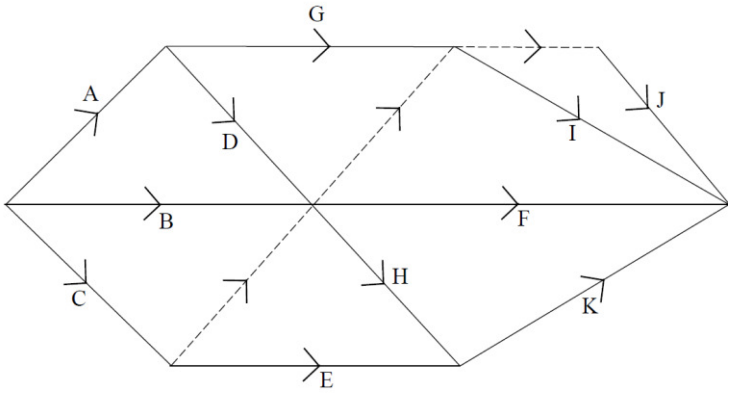
(c) B1: Correct calculation seen then 3 – an answer of 3 with no working scores B0

(d) M1: Not a cascade chart. 4 'workers' used at most and at least 10 different activities placed

A1: 4 workers. All 13 activities present (just once – so if an activity appears for two different workers and is happening at the same time this is A0). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

A1: 4 workers. All 13 activities present (just once). No errors

Activity	Duration	Time interval	IPA
A	8	0 -8	-
B	6	0 – 17	-
C	7	0 – 14	-
D	11	8 – 19	A
E	5	8 – 25	A
F	8	8 – 25	A, B, C
G	10	7 – 25	C
H	11	7 – 25	C
I	6	19 – 32	D
J	4	19 – 32	D
K	6	19 – 25	D
L	7	25 – 32	E, F, G, H, K
M	6	18 – 32	H

Qu	Scheme	Marks	AOs
5(a)	e.g. 	M1 A1 A1 A1 A1	 1.1b 1.1b 1.1b 1.1b 1.1b
		(5)	
(b)	Critical path: ADHK	B1	2.2a
		(1)	
			(6 marks)

Notes for Question 5

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. B, C and D labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity G is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only

(a) M1: At least nine activities (labelled on arc), one start, at least two dummies placed

A1: Activities A, B, C, D, E, G dealt with correctly

A1: Activities F, H and first two dummies + arrows dealt with correctly (the first two dummies are those that are required at the event at the end of activity B)

A1: Activities I, J and K dealt with correctly (note that I and J can start directly after the end of G)

A1: CSO – Final dummy + arrow, all arrows present for every activity with one finish and no additional dummies. Note that this is not a unique solution e.g. I, J could be interchanged, or the dummy could come after I or J, F and K could lead into the dummy etc. so please check these carefully. **Please check all arcs carefully for arrows – if there are no arrows on dummies then M1A1max**

Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

(b) B1: CAO (ADHK only)

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>6(a)</td><td>Simplex can only be applied when the non-negativity constraints are $\leqslant$</td><td>B1</td><td>3.5b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$3x+y+2z\le 30 \Rightarrow 3x+y+2z+s_1=30$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$x-y+z\ge 8 \Rightarrow x-y+z-s_2+a_1=8$</td><td>B1</td><td>2.5</td></tr>
<tr><td></td><td>$4y+2z\ge 15 \Rightarrow 4y+2z-s_3+a_2=15$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>$P=2x+2y-z \Rightarrow P=2x+2y-z-M(a_1+a_2)$ together with
$a_1+a_2=23-x-3y-3z+s_2+s_3$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$P-(2+M)x-(2+3M)y-(-1+3M)z+Ms_2+Ms_3=-23M$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>
<table>
<tr><td>b.v</td><td>x</td><td>y</td><td>z</td><td>s_1</td><td>s_2</td><td>s_3</td><td>a_1</td><td>a_2</td><td>Value</td></tr>
<tr><td>s_1</td><td>3</td><td>1</td><td>2</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>30</td></tr>
<tr><td>a_1</td><td>1</td><td>-1</td><td>1</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>8</td></tr>
<tr><td>a_2</td><td>0</td><td>4</td><td>2</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>1</td><td>15</td></tr>
<tr><td>P</td><td>$-(2+M)$</td><td>$-(2+3M)$</td><td>$-(3M-1)$</td><td>0</td><td>M</td><td>M</td><td>0</td><td>0</td><td>$-23M$</td></tr>
</table>
</td><td>M1
A1</td><td>3.3
2.2a</td></tr>
<tr><td></td><td></td><td>(7)</td><td></td></tr>
<tr><td>(c)</td><td>$s_1=26.25, a_1=11.75, y=3.75, x=z=s_2=s_3=a_2=0$</td><td>B1</td><td>3.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>The solution after the 1st iteration is not feasible because $a_1=11.75$ is an artificial variable which must be zero in a feasible solution</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(e)</td><td>The most negative value in the objective row is $2-1.5M$ so the pivot is a value from the z-column</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td>The 0.5 in the y row is the pivot because $\frac{3.75}{0.5}$ is less than both $\frac{26.25}{1.5}$ and $\frac{11.75}{1.5}$</td><td>dB1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Notes for Question 6

(a)

B1: CAO – e.g. not all of the constraints are $\leq$, the origin is not a (basic feasible) solution of the LP

(b)

B1: CAO $3x+y+2z+s_1=30$ (may be seen in the simplex tableau – allow any s_i (or s) for s_1)

B1: CAO $x-y+z-s_2+a_1=8$ (may be seen in the simplex tableau – allow any consistent s_i for s_2 (or t say) but not the same s_i as in the previous mark and allow any a_i for a_1)

B1: CAO $4y+2z-s_3+a_2=15$ (may be seen in the simplex tableau – same conditions as above)

M1: setting up the new objective which must be $P=2x+2y-z-M\left(a_1+a_2\right)$ and substituting for their a_1 and a_2 (if no working then the **correct** objective line in the tableau implies this mark)

A1: CAO $P-\left(2+M\right)x-\left(2+3M\right)y-\left(-1+3M\right)z+Ms_2+Ms_3=-23M$ (any equivalent form – need not be factorised and does not need to be re-arranged into this form - if no working then the **correct** objective line in the tableau implies this mark)

M1: setting up initial tableau – all four rows complete with two correct rows (but ignore b.v. column for this mark)

A1: CAO (any equivalent correct form)

(c)

B1: CAO $s_1=26.25, a_1=11.75, y=3.75, x=z=s_2=s_3=a_2=0$ (ignore expression for P if given)

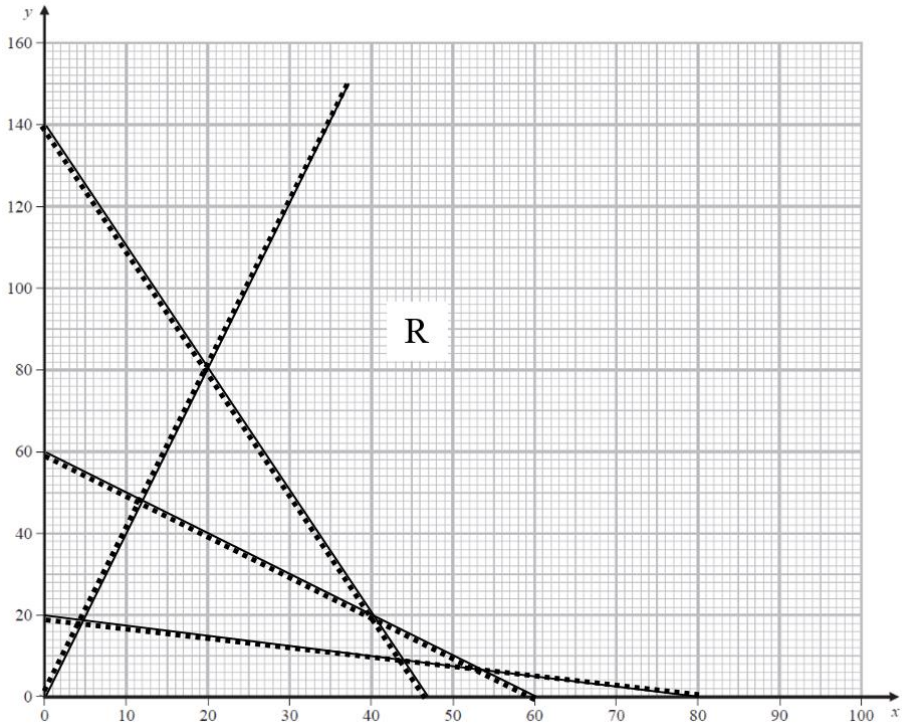
(d)

B1: correct reasoning of why the solution is not feasible e.g. a_1 is not zero but B0 for just stating that the artificial variable is non-zero (so must see either a_1 or 11.75 being stated as non-zero)

(e)

B1: correct reasoning of why the pivot comes from a value from the z-column so must say that the most negative value (in the objective row) is $2-1.5M$ (or this expression clearly implied)

dB1: correct justification of why the 0.5 in the third row is the next pivot (dependent on previous B mark) – so must compare or state that $\frac{3.75}{0.5}$ or 7.5 is less than both $\frac{26.25}{1.5}$ or 17.5 and $\frac{11.75}{1.5}$ or 7.8(3333….) – just stating that the 0.5 in the third row is the next pivot without reasoning is no marks in this part

Qu	Scheme	Marks	AOs
7(a)	$x+y\geq 60$ $y\leq \frac{4}{5}(x+y)$	B1 B1	3.3 3.3
		(2)	
(b)		B1 B1 B1 B1	1.1b 1.1b 1.1b 2.2a
		(4)	
(c)	objective line drawn or point-testing	M1 A1	3.1a 1.1b
	(20, 80) so 20 analogue watches and 80 digital watches	A1	3.2a
		(3)	
(d)	$20a+80d=4\,455$	B1ft	3.1b
	$a=5d$	B1	2.1
	Leading to $a=123.75$ and $d=24.75$ so an analogue watch costs £123.75 and a digital watch costs £24.75	dB1	2.2a
		(3)	
			(12 marks)

Notes for Question 7

(a) B1: CAO – allow any equivalent form of $x+y\geq 60$ - do not condone strict inequality

B1: CAO – allow any equivalent form of $y\leq \frac{4}{5}(x+y)$ (but not $y\leq 80\%(x+y)$ only) and need not be simplified - do not condone strict inequality – isw if correct answer is incorrectly simplified

In **(b)**, lines must be long enough to define the correct feasible region and would pass if extended through one small square of the points stated:

$x+y=60$ must pass within one small square of its intersection with the axes – (0, 60) and (60, 0)

$y+3x=140$ must pass within one small square of its intersection with the axes – (0, 140) and $\left(\frac{140}{3},0\right)$ (so at 46.666…, 0)

$4y+x=80$ must pass within one small square of its intersection with the axes – (0, 20) and (80, 0)

$y=4x$ must pass within one small square of (0, 0) and (25, 100)

In (b) condone for full marks lines which are drawn as dashed rather than solid

(b) B1: 2 lines drawn correctly

B1: 3 lines drawn correctly

B1: 4 lines drawn correctly

B1: Region, *R*, correctly labelled – not just implied by shading – dependent on scoring the first three marks in this part

(c) M1: Drawing the correct objective line (with gradient – 5) or its reciprocal (with gradient $-\frac{1}{5}$). Line must be correct to within one small square if extended from axis to axis. If lines shorter than (5, 0) to (0, 25) or (0, 5) to (25, 0) then M0. Or point testing at least two exact coordinates of their *R* using their objective function which must be of the form $k(5x+y)$ or $k(x+5y)$ for some positive real value *k*

A1: Correct objective line – condone lack of labelling of the objective line. Or point testing at least two of the correct exact coordinates which are (20, 80), (40, 20), (80, 0) and $\left(\frac{160}{3},\frac{20}{3}\right)$ using a correct objective function of the form $k(5x+y)$

A1: Correct number of watches – **must be in context** (and not just in terms of *x* and *y*) – dependent on a correct feasible region in **(b)** (so must have scored the first three marks in **(b)** but may not have labelled the FR as *R*)

Condone use of *x* for *a* and *y* for *d* in part (d)

(d) B1ft: A 'correct' equation (e.g. $20a+80d=4\,455$) involving their optimal point from **(c)** (accept any values even if non-integer) and 4455 –- note that for those who have done point testing in **(c)** the calculation 4455 / (their value for *P*) where *P* = $5x+y$ or $x+5y$ using their optimal point implies this mark

B1: CAO on the relationship between the costs of the two types of watches (*a* = 5*d*) – this mark may be implied e.g. 20(5*d*) + 80*d* = 4455 would score the first two marks in this part – note that for those who have done point testing in **(c)** the calculation 4455 / (their value for *P*) where *P* = $5x+y$ using their optimal point implies this mark e.g. just seeing 4455 / 180 is the first two marks in this part

dB1: CAO (dependent on first two B marks) – this mark is dependent on having the correct optimal point (20, 80) and is dependent on a correct feasible region in **(b)** (so must have scored at least the first three marks in **(b)**) – allow for $a=123.75$ and $d=24.75$ (so does not need to be in context or

units) – the correct answers with no working scores no marks in this part (however, note that 4455 / 180 is the minimum amount of working that is acceptable)

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